**In-Lab Group Activity for Week 7: Eigenvalues and Eigenvectors**

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***first     last***

**Problem 1: Diagonalization -** The eigenvectors for the matrix can be chosen to be

and

**a.** Find the eigenvalue that matches the eigenvector . **Hint**: Apply to each eigenvector:

**b.** Find the eigenvalue that matches the eigenvector .

**c.** Give the **matrix *V* of eigenvectors** *V*:

and the matching **matrix of eigenvalues** *D***:**

**d.** Find the determinant of the matrix *V* of eigenvectors, and then give its inverse .

-2

**e.** **Diagonalization and Powers**: Since , we can also write and hence any power of *A* such as can be found using .

Clearly is one million. Find (by hand) an exact expression for using the result . Show all details!

**Problem 2: The Dog Ate My Homework!**

The family dog chewed on an old homework assignment you are trying to review before an exam. The bottom row of the matrix *A* is missing, so let's represent it by symbols:

**CHOMP!**

The chewed-up homework still shows the eigenvectors for *A* are and

**a.** Find the eigenvalues and for each eigenvector. Both are integers.

**Hint**: Apply the fundamental identity to each eigenvector:

Similarly, for the second eigenvector we find:

**b.** Combine your two equations into a linear system for the missing values *a* and *b* and then give the complete matrix*.*

a+b=10

a-b=-10

a=-5

b=5

**System: Solution (for *a* and *b*):**

Does this matrix look familiar?

**c.** Above you obtained a system with two linear equations for the two unknowns *a* and *b*. Let's collect them into a vector and write your system in the matrix form . Give the matrix *M* and the vector .

**Problem 3: Magic Matrices in MATLAB**: Let denote the magic matrix returned by MATLAB where *n* is a positive integer. Here are a few.

**Exception for**

X

,     ,

Every number from 1 to appears exactly once in the magic matrix . Further, every column sum, row sum and diagonal sum is the same. These properties do **not** all hold for magic(2) which is an **exception**, so we will toss it out for the purposes of this exploration! Using MATLAB's **eig** command, explore the following properties for magic matrices **magic(n)** with *n* varying from 3 to 10. Remember we are excluding the case . Test the code below with different values of n in the range from 3 to 10.

clear, clc

n = 8 % size of magic matrix

M = magic(n)

[v, d] = eig(M) % eigenvectors and eigenvalues

**a.** The largest positive **eigenvalue** for magic(n) is:

**i.** n **ii.** **iii.** **iv.**

**b.** The **eigenvector** corresponding to the largest eigenvalue of can be chosen to be:

**i.** **ii.** **iii.** **iv.**

Each column vector above has *n* components.

**c.** **Explore**! Excluding the largest eigenvalue, if is an eigenvalue then so is .

Check for all *n* in the range from 3 to 10. If false, give n for your exception.

**i.** True **ii.** False when n = \_ \_

**d.** **Explore**! What is the multiplicity of the eigenvalue for the case of ?

**i.** 1 **ii.** **iii.** **iv.** **v.**

**e.** What is the **nullity** (dimension of the null space) for the case of ?

**i.** 1 **ii.** **iii.** **iv.** **v.**